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LETTER TO THE EDITOR

On the dynamic localization in 1D tight-binding systems

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Abstract. The dynamic localization of electrons induced simultaneously by DC and AC electric fields in the classical limit is investigated. Scattering processes are taken into account. It is shown that the localization induced by one field can be suppressed by another one when $n\Omega = \omega$ (ω is the Stark frequency and Ω is the AC field frequency). The localization is also effectively destroyed by scattering processes.

It is well known that, in the uniform electric field $\boldsymbol{E}$, the electron motion in a periodic potential undergoes Bloch oscillations with the Stark frequency $\omega = eEa$ (where a is the period of the potential and $\hbar = 1$). In the quantum limit, when $\omega > \Delta$ (Δ is the allowed band width), the Bloch oscillations manifest themselves in the form of discrete eigenstates in the electron energy spectrum (Stark ladder). The wavefunction of the Stark-ladder states is found to be localized (see for example [1, 2] and references therein). This phenomenon is called dynamic localization.

In the classical limit, when $\omega \ll \Delta$, these discrete eigenstates do not manifest themselves; however, electron motion remains finite. Their trajectories are confined to some region with the characteristic size $L \sim \Delta/eE$. In this sense this phenomenon is also called dynamic localization [3, 4]. A similar situation can arise in the AC electric field. For this localization the ratio between the field amplitude F and field frequency Ω has to satisfy the additional condition [3]

$$J_0(\omega_F/\Omega) = 0 \qquad (\omega_F = eFa \ll \Delta) \tag{1}$$

where $J_n(z)$ is the Bessel function of nth order. The behaviour of electrons in the classical limit can be treated semiclassically, i.e. described by the classical equation of motion, by the classical kinetic equation, etc. Note that in [3, 4] the dynamic localization was considered in the collisionless limit, where the collision frequency $\nu = \tau^{-1} \to 0$ (τ is the scattering time).

In this paper we consider the dynamic localization of electrons induced simultaneously by DC $\boldsymbol{E}$ and AC $\boldsymbol{F}(t) = \boldsymbol{F}\sin(\Omega t + \varphi)$ electric fields (φ is the initial phase of field). We will consider the one-dimensional tight-binding systems, where the electron energy spectrum has the form

$$\varepsilon_k(t) = \Delta \cos k(t)a \tag{2}$$

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As is well known, this model appropriately describes the electron behaviour in quantum wires or in one-dimensional superlattices. As in the [3,4] we restrict our investigation to the classical limit, when ω and $\omega_F \ll \Delta$ and the electron behaviour can be described by the classical equation of motion. The scattering processes will be taken into account. It will be shown that, even in the collisionless limit, the localization induced by one field can be suppressed by another one when $\omega = n\Omega$ (Stark resonance). The localization can be also effectively destroyed by scattering processes. Note that similar effects are well known in the theory of statistics localization [5].

To be specific we start with a well localized state at a particular site in the system at $t = 0$ and obtain the time evolution of the wavefunction or probability propagator $\psi_0(t)$. We say that localization takes place if, for large times, the probability propagator at this site remains finite. In contrast, if this amplitude goes to zero we conclude that diffusion takes place in the system. Similarly, the localization will take place if, for large time, the mean square displacement $\langle m^2 \rangle = \sum_{n=0}^{\infty} \psi_n(t)$ remains finite. If the mean square displacement increases without bound the localization is destroyed. Following the method proposed in [4] we start from the equation for the expansion coefficients $C_k(t)$ of the wavefunction

$$C_k(t) = C_k(0) \exp\left[-\mathrm{i} \int_0 t\varepsilon_k(t')\,\mathrm{d}t' \right]. \tag{3}$$

The electron behaviour is described by the classical equation of motion:

$$\frac{\mathrm{d}k}{\mathrm{d}t} = eE + eF\sin(\Omega t + \varphi) - \frac{k - k_0}{\tau} \tag{4}$$

where $k_0 = k(t = 0)$. The solution of equation (4) has the form

$$\begin{aligned} k = k_0 + eE\tau(1 - \mathrm{e}^{-t/\tau}) &+ \frac{eF\tau}{1 + \tau^2\Omega^2}(\sin(\Omega t + \varphi) - \mathrm{e}^{-t/\tau}\sin\varphi) \\ &+ \frac{eF\tau^2\Omega}{1 + \tau^2\Omega^2}[\mathrm{e}^{-t/\tau}\cos\varphi - \cos(\Omega t + \varphi)] \end{aligned} \tag{5}$$

Substituting equations (2) and (5) into (3) and performing the relevant calculations as in [4] we obtain the resulting expression for the probability propagator $\psi_m(t)$ and mean square displacement $\langle m^2 \rangle$ as follows:

$$\psi_m(t) = J_m^2\left(\Delta\sqrt{u^2(t) + v^2(t)}\right) \tag{6}$$

$$\langle m^2 \rangle = \tfrac{1}{4}\Delta^2(u^2(t) + v^2(t)) \tag{7}$$

where

$$u(t) = \int_0^t \cos(b(t'))\,\mathrm{d}t' \tag{8}$$

$$v(t) = \int_0^t \sin(b(t'))\,\mathrm{d}t' \tag{9}$$

and

$$b(t) = \omega\tau(1 - e^{-t/\tau}) + \frac{\omega_F \tau}{1 + \Omega^2\tau^2}(\sin(\Omega t + \varphi) - e^{-t/\tau}\sin\varphi) + \frac{\omega_F \Omega\tau^2}{1 + \Omega^2\tau^2}(e^{-t/\tau}\cos\varphi - \cos(\Omega t + \varphi)). \quad (10)$$

First let us consider the collisionless regime ($\nu = \tau^{-1} = 0$). In this limit the function $b(t)$ assumes the form

$$b(t) = \omega t - \frac{\omega_F}{\Omega}\cos(\Omega t + \varphi). \quad (11)$$

By using the following identities:

$$\cos(a \pm b\cos z) = \cos(a)J_0(b) + 2\sum_{n=1}^{\infty} J_n(b)\cos\left(a \pm \frac{n\pi}{2}\right)\cos nz \quad (12)$$

$$\sin(a \pm b\cos z) = \sin(a)J_0(b) + 2\sum_{n=1}^{\infty} J_n(b)\sin\left(a \pm \frac{n\pi}{2}\right)\cos nz \quad (13)$$

we get the following expressions for the functions $u(t)$ and $v(t)$:

$$u(t) = \frac{1}{\omega}\sin(\omega t)J_0\left(\frac{\omega_F}{\Omega}\right) + 2\sum_{n=1}^{\infty} J_n\left(\frac{\omega_F}{\Omega}\right) \times \left\{\frac{\cos[\frac{1}{2}(\omega + n\Omega)t + n(\varphi - \frac{\pi}{2})]\sin[\frac{1}{2}(\omega + n\Omega)t]}{\omega + n\Omega} + \frac{\cos[\frac{1}{2}(\omega - n\Omega)t - n(\varphi + \frac{\pi}{2})]\sin[\frac{1}{2}(\omega - n\Omega)t]}{\omega - n\Omega}\right\} \quad (14)$$

$$v(t) = \frac{1}{\omega}(\cos\omega t - 1)J_0\left(\frac{\omega_F}{\Omega}\right) + 2\sum_{n=1}^{\infty} J_n\left(\frac{\omega_F}{\Omega}\right) \times \left\{\frac{\sin[\frac{1}{2}(\omega + n\Omega)t + n(\varphi - \frac{\pi}{2})]\sin[\frac{1}{2}(\omega + n\Omega)t]}{\omega + n\Omega} + \frac{\sin[\frac{1}{2}(\omega - n\Omega)t]\sin[\frac{1}{2}(\omega - n\Omega)t - n(\varphi + \frac{\pi}{2})]}{\omega - n\Omega}\right\}. \quad (15)$$

Equations (6), (7), (14) and (15) recover the results of [3] and [4] in the limit $E = 0$ and $F = 0$ respectively. Equations (14) and (15) show that, generally, $u(t)$ and $v(t)$ are bounded oscillatory functions of time with all frequencies $\omega \pm n\Omega$ ($n = 0, 1, 2\ldots$). However, when $\omega = n\Omega$ ($n = 1, 2, 3\ldots$) these functions contain terms which are linearly proportional to the time t. For large time ($t \gg \Omega^{-1}$) these terms give the main contributions to $u(t)$ and $v(t)$ and they make $\psi_m(t) \to 0$ and $\langle m^2\rangle \to \infty$. The dynamic localization in this case is completely destroyed. For example, for $\omega = \Omega$ and $t \gg \Omega^{-1}$ one gets

$$u(t) \simeq J_1(\omega_F/\Omega)t\sin\varphi \quad (16)$$

$$v(t) \simeq -J_1(\omega_F/\Omega)t\cos\varphi \quad (17)$$

$$\psi_m(t) \simeq J_m^2[J_1(\omega_F/\Omega)\Delta t] \to 0 \quad (18)$$

$$\langle m^2\rangle \simeq \tfrac{1}{4}\Delta^2 J_1^2(\omega_F/\Omega)t^2 \to \infty. \quad (19)$$

Note that, though $u(t)$ and $v(t)$ are dependent on the initial phase φ, the probability $\psi_m(t)$ and mean square displacement $\langle m^2\rangle$ do not depend on it. In figure 1 the mean square displacement as a function of the dimensionless time Ωt for several values of $\mu = \omega/\Omega$ is depicted. We can see that for $\mu = \sqrt{1}, \sqrt{3}$, the mean square displacement remains a bounded function while for $\mu = 1, 2$, these curves rise to infinity. For computational calculations the ratio ω_F/Ω was set equal 2.405 as is required for the usual localization [3], and for simplicity φ was set equal to zero. This remarkable effect of suppressing the localization produced by a DC field by the superposition of an AC field can be understood by taking into account the fact that both harmonic motions come to resonance when $\omega = n\Omega$. Naturally for higher harmonics this effect is weaker as can be seen in figure 1.

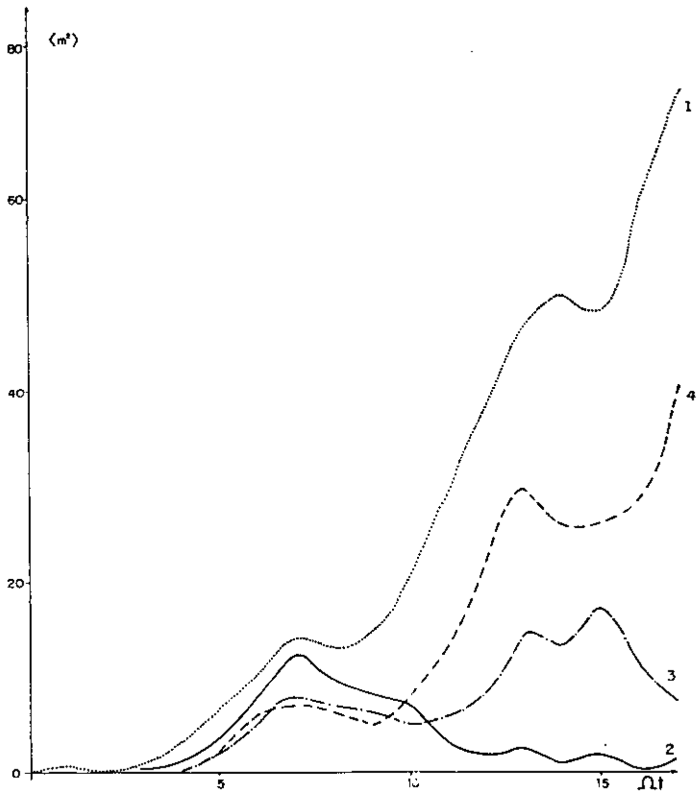

Figure 1. The mean square displacement for the following parameters: $\Delta = 4\Omega$, $\omega_F/\Omega = 2.405$, $\varphi = 0$; curve 1, $\mu = 1$; curve 2, $\mu = \sqrt{2}$; curve 3, $\mu = \sqrt{3}$; curve 4, $\mu = 2$.

Taking account of scattering processes ($\nu \neq 0$) the analytical solution can be obtained in the case of DC electric field ($F = 0$). In this case we have

$$b(t) = \omega\tau(1 - e^{-t/\tau}) \tag{20}$$

$$u^2(t) + v^2(t) = \tau^2\{[\mathrm{Si}(\omega\tau e^{-t/\tau}) - \mathrm{Si}(\omega\tau)]^2 + \mathrm{Ci}(\omega\tau e^{-t/\tau}) - \mathrm{Ci}(\omega\tau)]^2\} \tag{21}$$

where $\mathrm{Si}(z)$ and $\mathrm{Ci}(z)$ are the *sine* and *cosine* integrals. In the strong-scattering limit: $\omega\tau \ll 1$ from equations (6), (7) and (21) we have ($t \gg \tau$)

$$\psi_m(t) \simeq J_m^2(t\Delta) \tag{22}$$

$$\langle m^2\rangle \simeq \tfrac{1}{4}\Delta^2 t^2. \tag{23}$$

These results show that in the strong-scattering limit the probability propagator $\psi_m(t)$ goes to zero and the mean square displacement $\langle m^2 \rangle$ goes to infinity. The localization is suppressed. These results coincide with the results for the field-free case ($E = 0$) [3, 4]. This fact shows that in the strong-scattering regime, as in the field-free case, electric current cannot flow through the sample. In figure 2 we show the mean square displacement as a function of the dimensionless time $t\omega$ for several values of $\xi = \omega\tau$. We can see that on decreasing ξ (increasing the scattering) the localization is suppressed.

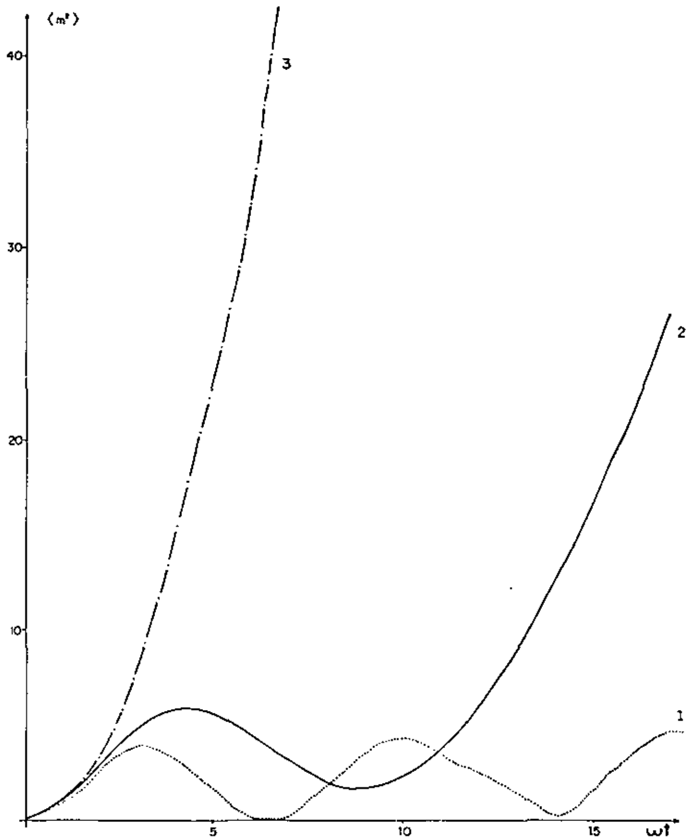

Figure 2. The mean square displacement for the following parameters: $\Delta = 4\omega$; curve 1, $\xi = 100$; curve 2, $\xi = 10$; curve 3, $\xi = 1$.

A similar situation arises in the case of an AC electric field ($E = 0$). The mean square displacement as a function of Ωt for several values $\zeta = \Omega\tau$ is depicted in figure 3. As in figure 1, the ratio ω_F/Ω is set equal to 2.405 and $\varphi = 0$.

To conclude the present work we note that 1D systems such as quantum wires and superlattices of very high quality can be grown, so theoretical models like the one presented here can be applied for real systems. At the same time non-linear phenomena like Bloch oscillations and negative differential conductivity [6] would be observed experimentally. In this work we have considered the general case of superposition of DC and AC electric fields as well as allowing for scattering to take place. As far as we know this is the first time that such a complete treatment of dynamic localization in 1D has been presented. Finally we would like to point out that there is a revival in interest in this subject, which can be measured by the number of papers coming out recently [7–9].

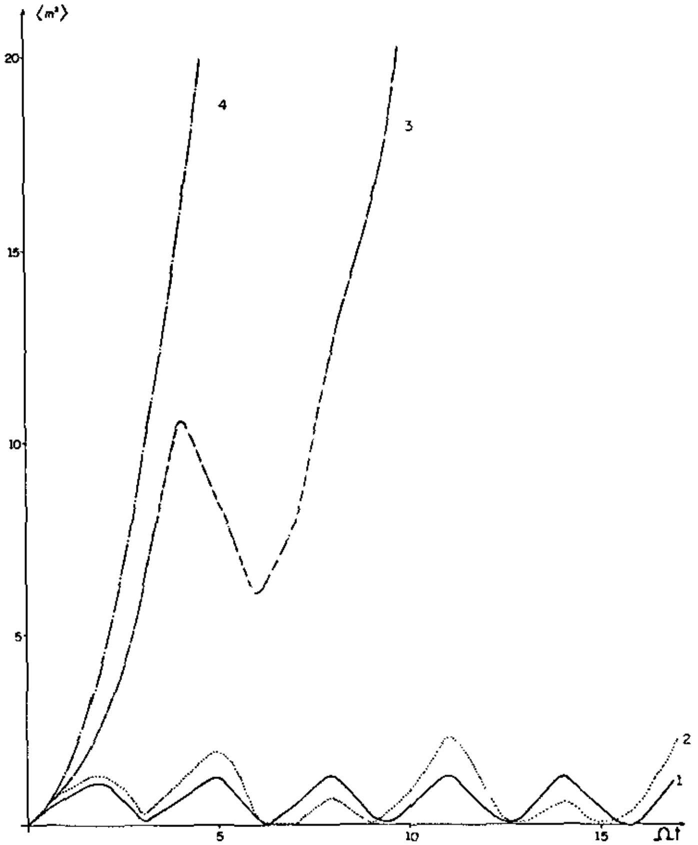

Figure 3. The mean square displacement for the following parameters: curve 1, $\zeta = 100$; curve 2, $\zeta = 10$; curve 3, $\zeta = 1$; curve 4, $\zeta = 0.1$. Other parameters are as in figure 1.

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References

[1] Roy C L and Mahapatra P K 1982 *Phys. Rev.* B **25** 1046
[2] Luban M and Luscombe J H 1986 *Phys. Rev.* B **34** 3674
[3] Dunlap D H and Kenkre V M 1986 *Phys. Rev.* B **34** 3625
[4] Nazareno H N and Gallardo J C 1989 *Phys. Status Solidi* b **153** 179
[5] Altshuler B L, Aronov A G and Khmenitsky D E 1981 *Solid State Commun.* **39** 619
[6] Beltram F, Capasso F, Sivco D L, Hutchinson A L, Chu S-N G and Cho A Y 1990 *Phys. Rev. Lett.* **64** 3167
[7] Soucail B, Ferreira R, Bastard G and Voisin P 1991 *Europhys. Lett.* **15** 857
[8] Holthaus M 1992 *Phys. Rev. Lett.* **69** 351
[9] Zhao X-G 1992 *J. Phys.: Condens. Matter* **4** L383